

Auctions, Bidding, and Contests

IT WASN'T THAT long ago that the typical image of an auction was that of an auctioneer with a snooty British accent calling out to a hushed room of bejeweled art collectors sitting in Louis XIV chairs and tugging at their ears to bid. With eBay, auctions have become just a touch more democratic.

The most familiar auction is where an item is put up for sale and the high bidder wins. At Sotheby's, it is a painting or an antique. On eBay, it is a Pez dispenser, a used drum set, or almost anything (except a kidney). On Google and Yahoo!, auctions for ad positions next to keyword searches bring in well over \$10 billion. In Australia, even houses are sold via auctions. The common denominator is that we have one seller and many buyers. The buyers compete against each other to gain the object and the high bidder wins.

The view of an auction as a way to sell something is too narrow. Auctions are also used to buy items. A good illustration is where a local government wants to build a road and takes bids to determine who will build the road. Here the winning bidder is the one who makes the *lowest* bid, as the government wants to buy the paving service as cheaply as possible. This is called a procurement auction. There's one buyer and many sellers to get the buyer's business.*

Bidding in an auction requires a strategy—though many people think all they need is a paddle. That leads to problems when people bid based on emotion or excitement. They live to regret it. To do well in an auction setting requires a strategy. Should you bid early or wait until the auction is almost over and then jump in? If you value an item at \$100, how high should you bid? How do you avoid winning the auction but then regretting that you've overpaid? As we have discussed before, this phenomenon is known as the winner's curse; here we'll

explain how to avoid it.

Should you even bid in the auction? The house auction market in Australia illustrates the buyer's dilemma. Imagine that you are interested in a house that is due to be auctioned on July 1. But there's a house you like even more that will be auctioned off a week later. Do you wait to bid in the second auction and risk ending up with neither?

Our plan is to start with a description of some basic auction types and then discuss how game theory can help you bid—and know when not to.

ENGLISH AND JAPANESE AUCTIONS

The most famous type of auction is known as the English or ascending auction. In this format, the auctioneer stands at the front of the room calling out ever-increasing bids:

Do I hear 30? 30 from the lady in the pink hat.

40? Yes, 40 from the gentleman on my left.

Will someone bid 50? 50, anyone?

40 going once, going twice, sold.

Here the optimal bidding strategy, although it hardly merits the term strategy, is simple. You bid until the price exceeds your value and then you drop out.

There is often a bit of trickiness over the issue of bidding increments. Imagine that the bidding goes up in units of 10 and your value is 95. Then you would stop bidding at 90. Of course, knowing that, you might want to think about whether you should be the high bidder at 70 or 80, recognizing that 90 will be your last bid. In the discussion that follows, we will assume that the bidding increments are very small, say a penny, so that these endgame issues are not important.

The only hard part is determining what is meant by your “value.” What we mean by your value is your walkaway number. It is the highest price at which you still want to win the item. At a dollar more you would rather pass, and at a dollar less you are willing to pay the price, but just barely. Your value might include a premium you put on not having the item fall into a rival's hands. It

could include the excitement of winning the bidding. It could include the expected resale value in the future. When all of the components are put together, it is the number such that if you had to pay that price, you no longer care if you've won or lost the auction.

Values come in two flavors, private and common. In a world of private values, your value for the item doesn't depend at all on what others think it is worth. Thus your value of a personalized signed copy of *The Art of Strategy* doesn't depend on what your neighbor might think it is worth. In a common value situation, the bidders understand that the item has the same value for all of them, although each might have a different view as to what that common value is. A standard example is bidding for an offshore oil lease. There is some amount of oil underground. While that amount might be unknown, it is the same whether Exxon or Shell wins the bidding.

In truth, the value of an item usually has elements of both private and common components. Thus one oil company might be better at extracting the oil than the other, which then adds a private value element to something that is mostly common.

In situations with a common value, your best guess as to the value of an item might depend on who else or how many others are bidding and when they drop out. An English auction keeps that information hidden, as you never know who else is willing to bid but hasn't yet moved. Nor are you sure when someone has dropped out. You know their last bid, but not how high they would have gone.

There is a variation of the English auction that is more transparent. In something called a Japanese auction, all of the bidders start with their hands raised or buttons pressed. The bidding goes up via a clock. The clock might start at 30 and then proceed to 31, 32,...and upwards. So long as your hand is raised, you are in the bidding. You drop out by lowering your hand. The catch is that once you lower your hand, you can't put it back up again. The auction ends when only one bidder remains.

An advantage of the Japanese auction is that it is always clear how many bidders are active. In an English auction, someone can remain silent even though they were willing to bid all along. The person can then make a surprise entry late in the contest. In the Japanese auction, you know exactly how many competitors there are and even the prices at which each drops out. The Japanese auction is thus just like an English auction where everyone has to reveal their hand.

The outcome of a Japanese auction is easy to predict. Since bidders drop out when the price hits their value, the last person remaining will be the one with the highest valuation. The price the winner will pay will equal the second highest valuation. The reason is that the auction ends at the moment the second-to-last

bidder drops out. The last price is the valuation of the second highest bidder.

Thus the item is sold to the person with the highest valuation, and the seller receives a payment equal to the second highest valuation.

VICKREY AUCTION

In 1961, Columbia University economist and future Nobel laureate William Vickrey developed a different type of auction. He called it a second-price auction, though we now call it a Vickrey auction in his honor.*

In a Vickrey auction, all the bids are placed in a sealed envelope. When the envelopes are opened to determine the winner, the highest bid wins. But there's a twist. The winner doesn't pay his or her bid. Instead, the winner only has to pay the second highest bid.

What is remarkable, even magical, about this auction is that all the bidders have a dominant strategy: bid their true valuation. In a regular sealed-bid auction where the high bidder wins and pays his actual bid, bidding strategy is a complicated problem. What you should bid depends on how many other bidders are in the game, what you think their value is for the item, even what you think they think your value is. The result is a complicated game where everyone has to consider what everyone else is doing.

In a Vickrey auction, all you have to do is figure out what the item is worth to you and then write down that amount. You don't need to hire a game theorist to help you bid, alas. Actually, we like that result. Our goal is to be strategic when designing a game so that the players don't have to be strategic when they play it.

The reason why your bidding strategy is so simple is that it is a dominant strategy. A dominant strategy is your best play no matter what others are doing. Thus you don't need to know how many others there are or what they are thinking or doing. Your best strategy doesn't depend on what anyone else bids.

This brings us to the question of how we know that bidding your valuation is a dominant strategy. The following example is the basis of the general argument.

You are in a Vickrey auction and your true value of the item is \$60. But instead of bidding \$60, you bid \$50. To show that this is a bad idea, we put on our consequentialist hat. When does bidding \$50 rather than \$60 lead to a different outcome? Actually, it is easier to turn this question around. When does bidding \$50 or \$60 lead to the same result?

TRIP TO THE GYM NO. 6

Imagine that you could find out how much the other bidders were submitting in a Vickrey auction before you had to put in your bid. Ignoring the ethical issues for a moment, how much would this be worth to you?

If someone else bids \$63 or \$70 or anything else above \$60, then both \$50 and \$60 are losing bids. Hence there is no difference between them. In both cases, you lose the auction and walk away with nothing.

The \$50 and \$60 bids also lead to identical (but this time happier) outcomes if the highest other bid is below \$50, say \$43. If you bid \$60, then you win and pay \$43. If you had bid \$50, you would also have won and paid \$43. The reason is that in both cases you are the highest bidder and what you pay is the second highest bid, which is \$43. Bidding \$50 doesn't save you any money (compared to bidding \$60) when the second highest bid is \$43 or anything below \$50.

We've looked at the cases in which the two bids lead to exactly the same result. Based on these cases, there is no reason to prefer one bid over the other. What is left is where the paths diverge. This is how we can judge which bid leads to a better result.

There's no difference if any rival bid is above \$60 or all are below \$50. The only remaining case is where the highest competitive bid is between \$50 and \$60, say \$53. If you bid \$60, then you will win and pay \$53. If you were to have bid \$50, then you would lose. Since your value is \$60, you would rather win and pay \$53 than lose.

Thus the only time a bid of \$50 leads to a different outcome than one of \$60 is when you lose the auction but you wished you had won. This demonstrates that you never want to bid \$50 or anything below your true value. A similar argument shows that you never want to bid more than your true value.

REVENUE EQUIVALENCE

At this point, you may have figured out that the Vickrey auction gets you to the same outcome as the English (or Japanese) auction, all in one step. In both cases, the person with the highest value ends up winning the auction. In both cases, what the winning bidder has to pay is the second highest valuation.

In the English (or Japanese) auction, everyone bids up to his or her value, so the auction ends when the bidding gets up to the second highest valuation. The

remaining bidder is the person with the highest value. And subject to the vagaries of the bidding interval, what the winning bidder pays is the bid at which the penultimate bidder drops out, namely the second highest valuation.

In the Vickrey auction, everyone bids his or her true value. Thus the person with the highest valuation is the winning bidder. According to the rules, that person only has to pay the second highest bid, which is just the second highest valuation.

Thus it appears that the two auctions get to exactly the same place. The same person wins and the winner pays the same price. Of course, there is always the issue of the bidding interval in that the person with a valuation of 95 might drop out at 90 if the bidding increment comes in 10s. But with small enough increments, the person will drop out just at the valuation.

There is one subtle difference between the two auctions. In the English auction, a bidder learns something about what others think the item is worth by seeing some of their bids. (There are many potential bids that are not seen.) In the Japanese variant, the bidders learn even more. Everyone sees where everyone drops out. In contrast, in the Vickrey auction, the winning bidder doesn't get a chance to learn anything about the other bids until the auction is all over. Of course, in a private value auction, a bidder doesn't care about what others think the item is worth. Thus the extra information is irrelevant. This allows us to conclude that in a private value setting the seller would make the same amount of money by employing a Vickrey auction or an English (or Japanese) auction.

It turns out that this is part of a much more general result. In many cases a change in the rules doesn't lead to any more or less revenue to the seller.

Buyer's Premium

If you win an auction at Sotheby's or Christie's, you might be surprised to learn that you owe more than you bid. This isn't just sales tax we are talking about. The auction houses tack on a 20 percent buyer's premium. If you win the auction with a \$1,000 bid, you will be expected to write them a check for \$1,200.

Who pays the buyer's premium? The obvious answer is the buyer. But if the answer were really that obvious, we surely wouldn't have asked the question—or would we, just to keep you honest?

Okay, it isn't the buyer who pays—it's the seller. To get this result all we need to assume is that the buyer is aware of this rule and takes it into account

when bidding. Put yourself in the position of a collector who is willing to pay \$600. How high will you bid? Your top bid should be \$500—as you can anticipate that saying \$500 really means that you have to pay \$600 after the buyer's premium.

You can think of the buyer's premium as being nothing more than a currency conversion or a code. When you say \$100, you really mean \$120.* Each bidder scales back his bid accordingly.

If your winning bid is \$100, you have to write a check for \$120. You don't care that the \$120 is divided up \$100 to the seller and \$20 to the auction house. You only care that the painting costs \$120. From your perspective, you can just as well imagine that the buyer receives the full \$120 and then turns over \$20 to the auction house.

Our point is that the winning bidder still pays the same amount. The only difference is that now the auction house gets some percentage of the total. Thus the cost is borne entirely by the seller, not the buyer.

The larger takeaway here is that you may change the rules of the game, but the players will adapt their strategies to take those new rules into account. In many cases, they will precisely offset what you've done.

ONLINE AUCTIONS

While the Vickrey auction may go all the way back to Goethe, it was still relatively uncommon until recently. It has now become the standard for online auctions. Take the case of eBay. You don't bid directly in an eBay auction. Instead, you do something called a proxy bid. You authorize eBay to bid up to your proxy. Thus if you give them a proxy bid of \$100 and the current high bid is \$12, eBay will first bid \$13 for you. If that bid is high enough to win, that's where they stop. But if someone else has put a proxy bid of \$26, then eBay will bid up to \$26 for that person and your proxy will go all the way up to \$27.

It would seem that this is just like a Vickrey auction. Think of the proxy bids as being like the bids in a Vickrey auction. The person with the highest proxy bid ends up being the winner and the amount the person pays is equal to the second highest proxy bid.

To make this concrete, imagine there are three proxy bids:

A: \$26

B: \$33

C: \$100

A's proxy will drop out once the bidding gets to \$26. B's proxy will force the bidding up to that level. And C's proxy will push the bidding all the way up to \$34. Thus C will win the auction and pay the second highest proxy bid.

If everyone had to submit their proxy bids at the same time and once and for all, the game truly would be the same as a Vickrey auction and we could advise everyone to play it straight and bid their true value. Bidding the truth would be a dominant strategy.

But the game isn't quite played that way, and these little hiccups lead people to get fancy with their bids. One complication is that eBay will often have several similar items for sale all at the same time. Thus if you want to buy a used Pearl Export drum set, you have a choice of ten or so to choose from at any one time. You might like to bid on whichever one is cheapest up to \$400. While you are willing to pay up to \$400 for any one of the sets, you wouldn't bid \$300 on one version while another might be bought at \$250. You might also prefer to bid in an auction that is closing sooner over one that ends in a week so you don't have to wait to know whether or not you've won.

What this comes down to is that your value of the item being sold depends on what else is up for sale, both now and in the future. Thus you can't place a valuation independent of the auction.

Sniping

Let's take a case where multiple items and issues of timing aren't a concern. Consider an auction for a one-of-a-kind item. Now is there any reason not to play it straight and enter your true value in your proxy bid?

As an empirical matter, people don't play it straight. They often wait until the last minute or even second before entering their best proxy bid. The name for this is *sniping*. Indeed, there are Internet services such as Bidnapper and others that will do the sniping for you so you don't have to wait around for the auction to end before submitting your bid.

Why snipe? We've shown that bidding your true value is a dominant strategy in a Vickrey auction. Sniping must arise because of the subtle differences between proxy bidding and a Vickrey auction. The key difference is that the other bidders might get to learn something from your proxy bid before the auction is over. If what they learn influences how they bid, then you can have an incentive to keep your bid, even your proxy bid, hidden.

An early proxy bid might give away valuable information. For example, if a furniture dealer bids on a specific Bauhaus chair, you might (quite reasonably) infer that the piece is authentic and of historical interest. If the dealer is willing to buy the chair at a price of \$1,000, then you would be happy to pay \$1,200, a better price than you could hope to get from that same dealer. Thus the dealer doesn't want others to know how high he or she is willing to go. That leads the dealer to wait until the very end before putting in a bid. At that point, it is too late for you or others to react. By the time you've discovered the dealer is bidding, the auction is over. Of course, that implies that the bidder's true identity is known to others and that the bidder can't come up with an alias.* Since sniping is so common, that suggests there are other explanations.

We think the best explanation for sniping is that many bidders don't know their own value. Take the case of a vintage Porsche 911. The bidding starts at \$1. Of course, we don't value the car at \$1. We value the car at \$100, even at \$1,000. Provided the bidding is below \$1,000 we can be confident that this is a great deal. We don't have to look up the Blue Book value or even speak to our spouse about the need for an extra car. The point here is that we are lazy. Figuring out our true value for an item takes work. If we can win the auction without ever having to go through that effort, we would prefer to take the shortcut.

This is where sniping comes into play. Imagine that our expert buyer values the vintage Porsche at \$19,000. The buyer would prefer to keep the bidding low for as long as possible. If the buyer enters a \$19,000 proxy at the start, then our mindless proxy bid of \$1,000 will push the price right up to \$1,000. At that point, we will learn that we need to get more information. In the process, our spouse might go along for the ride and let us bid up to \$9,000. That could push the final price up to \$9,000 or higher if other bidders get a chance to do their homework.

But if the \$19,000 proxy bidder keeps his or her powder dry, then the bidding may not escalate above \$1,000 until the last moments of the auction, at which point it is too late for us to reenter a higher bid, assuming we were even paying attention and could get the quick okay from the spouse to bid more.

The reason to snipe is to keep others in the dark about their own valuations. You don't want people to learn that their lazy bid doesn't have a chance to win. If they find out early enough, they will have a chance to do their homework, which can only lead to you having to pay more, if you still win.

BIDDING AS IF YOU'VE WON

A powerful idea in game theory is the concept of acting like a consequentialist. By that we mean to look ahead and see where your actions have consequences. You should then assume that situation is the relevant one at the time of your initial play. It turns out that this perspective is critical in auctions and in life. It is the key tool to avoid the winner's curse.

To make this concrete, imagine that you ask someone to marry you. The person can say yes or no. If the answer is no, then there's nothing for you to go through with. But if the answer is yes, then you are on your way to getting hitched. Our point is that you should presume the answer will be yes at the time you pop the question. We are well aware that this is taking an optimistic perspective. Your intended could say no and you will be very disappointed. The reason to assume that your intended will say yes is to be prepared for that outcome. In that case, you should be saying yes as well. If upon hearing that your intended says yes, you then want to reconsider, you shouldn't have asked in the first place.

In a marriage proposal, assuming the answer will be yes is a pretty natural way of going about things. In the case of negotiations and auctions, this is an approach that has to be learned. Try your hand at the following game.

ACME

You are a potential buyer for ACME. Because of your extensive knowledge of game theory, you will be able to increase ACME's value by 50 percent, whatever it is. The problem is that you have some doubts as to the current value. After completing your due diligence, you place the value at somewhere between \$2 million and \$12 million. The average value is \$7 million and your view is that all the options in the \$2 to \$12 million range are equally likely. The way the bidding is set up, you get to make a single take-it-or-leave-it bid to the owners. They will accept any bid that exceeds the current value and reject otherwise.

Say that you bid \$10 million. If it turns out that the company is presently worth \$8 million, then you can make it worth \$12 million. You will pay \$10 million for a company worth \$12 million, and so your profit will be \$2 million. If the company is only worth \$4 million, then you will make it worth \$6 million, but have paid \$10 million and thus end up \$4 million behind.

What is the most you can offer the current owners and still expect to break even? By break even we mean that you might not make money in every situation, but on average you'll neither make nor lose any money. Note that we don't recommend bidding this amount. You should always bid something less

that this amount. This is just a way of figuring out a cap on your bids.

When faced with this problem, most people reason as follows:

On average the company is worth \$7 million. I can make it worth 50 percent more, or \$10.5 million. Thus I can bid up to \$10.5 million and still not expect to lose money.

Is \$10.5 million where you came out? We hope not.

Think back to the marriage proposal. You've proposed an acquisition here. What if they say yes? Do you still want to go ahead? If you offer \$10.5 million and the owners say yes, then you've learned some bad news. You now know that the company is not worth \$11 million or \$12 million today. When the owners say yes to an offer of \$10.5 million, the company will be worth somewhere between \$2 million and \$10.5 million, or \$6.25 million on average. The problem is that even with your 50 percent increase in performance, that only brings the value up to \$9.375 million, well below the \$10.5 million you offered.

This is a serious problem. It appears that if they say yes, then you no longer want to buy the company. The solution to this problem is to presume that your offer will be accepted. In that case, if you were to offer \$8 million, you can predict that when it is accepted the company is worth between \$2 million and \$8 million, for an average value of \$5 million. A 50 percent premium on \$5 million only gets you up to \$7.5 million, not enough to justify the \$8 million offer.

An offer of \$6 million just does the trick. You can anticipate that when the seller says yes, the company is worth between \$2 million and \$6 million, for an average value of \$4 million. The 50 percent premium brings the value to you back up to \$6 million or breakeven. The fact that the seller says yes is bad news but not fatal to the deal. You have to adjust down your offer to take into account the circumstances under which a seller will say yes to you.

Let's put this all together. If you offer \$6 million and you presume that your offer will be accepted, then you anticipate that the company will only be worth \$4 million and you won't be disappointed when your offer is accepted.* Quite often your offer will be rejected, in which case you will have underestimated the value of the company, but in those cases you don't end up with the company, so the mistake doesn't matter.

This idea of presuming you've won is a critical ingredient to making the right bid in a sealed-bid auction.

SEALED-BID AUCTIONS

The rules of a sealed-bid auction are simple. Everyone puts his or her bid in a sealed envelope. The envelopes are opened and the high bidder wins and pays his or her bid.

The tricky part of a sealed-bid auction is determining how much to bid. For starters, you should never bid your valuation (or worse, something more). If you do so, you are guaranteed to break even at best. This strategy is dominated by shading your bid to some amount below your valuation. That way, at least you have a chance to come out ahead.[†] How much you should shade your bid depends on how many others are competing in the auction and what you expect others to bid. But what they bid depends on what they expect you to bid. The key step to cutting through this infinite loop of expectations is to always bid as if you've won. When putting down your bid, you should always assume that all of the other bidders are below you. And then with that assumption, you should ask if this is your best bid. Of course, you will often be wrong when making that assumption. But when you're wrong, it won't matter—others will have outbid you and so you won't have won the auction. But when you're right, you'll be the winning bidder and thus have made the correct assumption.

Here's a way of demonstrating that you should always bid as if you've won. Imagine that you have a confederate inside the auction house. The confederate has the ability to adjust your bid downward in the event that you have the highest bid. Unfortunately, he doesn't know the other bids and can't tell you precisely how much to lower your bid. And if you don't have the highest bid, there's nothing he can do to help you.

Would you want to employ his service? You might not because it is unethical. You might not because you are afraid of turning a winning bid into a losing bid. But play along and imagine that you are willing to use his services. Your original bid was \$100, and after learning that this was the winning bid, you instruct him to lower the bid to \$80.

If this was a good idea, you might as well have bid \$80 right from the start. Why? Let's compare the two cases.

Case A	Case B
Bid \$100	Bid \$80
Lower bid to \$80 if \$100 was highest	

If \$100 would have lost, then there's no difference between bidding \$100 or

\$80. Both would be losing bids. If \$100 would have won, then your confederate would lower the bid to \$80, in which case you will end up in the same place as if you had bid \$80 all along. In short, there is no advantage to bidding \$100 and then reducing it to \$80 (when you've won) compared to bidding \$80 from the start. Since you can get the same result without having the confederate and acting unethically, you might as well bid \$80 from the start. What this all says is that when you are thinking about how much to bid, you should pretend that all of the other bidders are somewhere below your bid. Armed with this assumption, you then consider your best bid.

We'll return to figuring out just how much to bid after making a short detour to the Netherlands.

DUTCH AUCTIONS

Stocks are traded on the New York Stock Exchange. Electronics are sold in Akihabara, Tokyo. Holland is where the world goes to buy flowers. At the Aalsmeer Flower Auction, the auction "house" takes up some 160 acres. On a typical day, some 14 million flowers and a million potted plants change hands.

What makes Aalsmeer and other Dutch auctions a bit different from Sotheby's is that the bidding goes in reverse. Instead of starting with a low price and having the auctioneer call out successively higher prices, the auction starts with a high price that declines. Imagine a clock that starts at a hundred and then winds down to 99, 98,... The first person to stop the clock wins the auction and pays the price at which the clock was stopped.

This auction is the reverse of the Japanese auction. In the Japanese auction, all of the bidders indicate their participation. The prices keep rising until only one bidder is left. In the Dutch auction, prices start high and fall until the first bidder indicates his or her participation. If you raise your hand in a Dutch auction, the auction stops and you've won.

You don't have to go to the Netherlands to participate in a Dutch auction. You could send an agent to bid for you. Think for a moment about the instructions you might give your agent. You might say to wait until the price of petunias falls to €86.3 and then bid. As you contemplate those instructions, you should anticipate that if the bidding ever gets down to €86.3, then you will be the winning bidder. If you were at the auction house, you'd know that all of the other bidders have yet to act. Armed with this knowledge, you don't want to change your bid. If you wait a moment longer, one of the other bidders might jump in and take your place.

Of course that is true all along. Anytime you wait, another bidder might jump in. The issue is that the longer you wait, the bigger the profit you risk losing. And the longer you wait, the greater the risk that one of the other bidders is about to jump in. At your optimal bid, the savings from paying a lower bid is no longer worth the increased risk of losing the prize.

In many ways this is similar to what you might do in a sealed-bid auction. The instruction you give your bidding agent is akin to what you would write down as your sealed bid. Everyone else does the same. The person who writes down the highest number is the same as the person who first raises his or her hand.

The only difference between a Dutch auction and a sealed-bid auction is when you bid in a Dutch auction, you know you've won. When you write down your bid in a sealed-bid auction, you only find out later if you've won or not. But remember our guidance. In a sealed-bid auction, you are supposed to bid *as if* you've won. You're supposed to pretend that all of the other bidders are somewhere below you. This is exactly the situation you are in when competing in a Dutch auction.

TRIP TO THE GYM NO. 7

How much should you bid in a sealed-bid auction? For simplicity, you can assume that there are only two bidders. You believe that the other bidder has a value that is equally likely to be anything between 0 and 100, and the other bidder has the same belief about you.

Thus the way you bid in the two auctions is identical. Just as an English auction and a Vickrey auction end up in the same place, so do a sealed-bid auction and a Dutch auction. Since participants bid the same amount, the sellers get the same amount. Of course, that doesn't yet tell us how much to bid. It just says that we have two mysteries with the same answer.

The answer for how much to bid comes from one of the most remarkable results in auction theory: the revenue equivalence theorem. It turns out that when the valuations are private and the game is symmetric, the seller makes the same amount of money on average whether the auction type is English, Vickrey, Dutch, or sealed-bid.* What that means is that there is a symmetric equilibrium to the Dutch and sealed-bid auctions where the optimal bidding strategy is to bid what you think the next highest person's value is given the belief that you have

the highest value.

In a symmetric auction, everyone has the same beliefs about everyone else. For example, everyone might think that each bidder's value is equally likely to be anything between 0 and 100. In this case, whether the auction is Dutch or a sealed-bid, you should bid what you expect the next highest bidder's value to be given that all of the other values are below your own. If your value is 60, you should bid 30 if there is only one other bidder. You should bid 40 if there are two other bidders and 45 if there are three other bidders.*

You can see that this would lead to revenue equivalence. In a Vickrey auction, the person with the highest value wins but only pays the second highest bid, which is the second highest valuation. In a sealed-bid auction, everyone bids what they think the second highest valuation is (given they are the highest). The person with the truly highest valuation will win and the bid will be the same on average as the result in a Vickrey auction.

The larger moral here is that you can write down a set of rules for a game, but the players can undo those rules. You can say that everyone has to pay twice their bid, but that will just lead people to bid half as much. You could say that people have to pay the square of their bids, but that will just lead people to square root what they would otherwise have done. That is ultimately what is going on in a sealed-bid auction. You can tell people that they have to pay their bid rather than the second highest bid. In response, they'll change what they write down. Instead of bidding their true value, they will shade their bid downward to the point where it equals what they expect the second highest value to be.

To see if you are a believer, try your new intuition out on the world's biggest auction, namely the market for T-bills.

T-BILLS

Each week, the U.S. Treasury holds an auction that determines the interest rate on the national debt, at least the part that comes due that week. Until the early 1990s, the way the auction worked was that the winning bidders paid their bids. After some prodding from Milton Friedman and other economists, the Treasury experimented with uniform pricing in 1992 and made the move permanent in 1998. (The secretary of the treasury at the time was Larry Summers, a distinguished economist.)

We'll explain the difference between two cases through an example. Imagine that the Treasury had \$100 million in notes to sell one week. There were ten bids

that came in at

Amount Bid at Interest Rate	Cumulative Amount
\$10 million at 3.1%	\$10 million
\$20 million at 3.25%	\$30 million
\$20 million at 3.33%	\$50 million
\$15 million at 3.5%	\$65 million
\$25 million at 3.6%	\$90 million
\$20 million at 3.72%	\$110 million
\$25 million at 3.75%	\$135 million
\$30 million at 3.80%	\$165 million
\$25 million at 3.82%	\$190 million

The Treasury wants to pay the lowest interest rate possible. That means they will start by first accepting the lowest bids. Thus all of the bidders who were willing to take 3.6% or below are winners along with half of the bidders who were willing to take 3.72%.

Under the old rule, the \$10 million bid at 3.1% would win and those bidders would get only 3.1% on their Treasury note. The \$20 million bid at 3.25% would be awarded notes paying 3.25% and so on all the way up to the \$20 million bid at 3.72%. Note that there is more bid at 3.72% than can be fulfilled with the \$100 million for sale so that only half that amount will be sold and the other half will walk away empty-handed.*

Under the new rule, all of the bids between 3.1% and 3.6% are winning bids, as are half of those bidding 3.72%. With the uniform price rule, everyone gets the highest rate of any winning bid, in this case 3.72%.

Your first reaction might be to think that the uniform pricing rule is much worse for the government (and better for investors). Instead of paying between 3.1% and 3.72%, the Treasury pays everyone 3.72%.

Based on the numbers used in our example, you'd be correct. The problem with this analysis is that people won't bid the same way in the two auctions. We used the same number only to illustrate the mechanics of the auction. This is the game theory analog of Newton's Third Law of Motion—for every action there is a reaction. If you change the rules of the game, you must expect that players will bid differently.

Let's take a simple example to drive this point home. Imagine that the Treasury had said that instead of getting the interest rate that you bid, you would get 1% less. So a bid of 3.1% would only pay 2.1%. Do you think that would

change how much interest they would have to pay?

If we stuck with the same eight bids as above, the answer is yes, as the 3.1% becomes 2.1% and the 3.25% becomes 2.25%, and so on. But under this new regime, anyone who had previously planned on bidding 3.1% would now bid 4.1%. Everyone would bid 1% higher, and after the Treasury adjustment, things would play out just as before.

Indeed, this takes us to the second part of Newton's Third Law: For every action, there is a reaction, equal and opposite. That latter part may also apply to bidding, at least for the cases we've looked at. The reaction of the bidders offsets the changes in the rules.

After bidders adjust their strategies, the Treasury should expect to pay the same interest rates using a uniform price rule, as when winners get paid their bid. But life is much easier for bidders. A bidder who is willing to accept 3.33% no longer has to strategize about whether to bid 3.6% or 3.72%. If they value the bonds at 3.33%, they can bid 3.33% and know that, if they win, they will get at least 3.33% and most likely something higher. The Treasury doesn't lose any money, and bidders have a much simpler job.*

Many games that might not at first look like an auction turn out to be one. We turn now to look at two battle of wills, the preemption game and the war of attrition. In both contests, the situation is much like an auction.

THE PREEMPTION GAME

On August 3, 1993, Apple Computer launched the Original Newton Message. The Newton was more than a flop. It was an embarrassment. The handwriting recognition software developed by Soviet programmers didn't seem to understand English. In a *Simpsons* episode, the Newton misinterpreted "Beat up Martin" as "Eat up Martha." Doonesbury cartoons lampooned the mistakes made by its handwriting recognition.



The Newton was scrapped five years later, on February 27, 1998. While Apple was busy failing, in March of 1996 Jeff Hawkins introduced the Palm Pilot 1000 handheld organizer, which quickly grew to a billion in annual sales.

The Newton was a great idea, but it was not ready for prime time. That's the paradox. Wait until you're fully ready and miss the opportunity. Jump in too soon and fail. The launch of *USA Today* faced this same issue.

Most countries have longstanding national newspapers. France has *Le Monde* and *Le Figaro*, England has *The Times*, the *Observer*, and the *Guardian*. Japan has the *Asahi Shimbun* and *Yomiuri Shimbun*, China has the *People's Daily*, and Russia has *Pravda*. India has *The Times*, the *Hindu*, *Dainik Jagran*, and some sixty others. Americans were alone in not having a national daily. They had national magazines (*Time*, *Newsweek*) and the weekly *Christian Science Monitor* but no national daily paper. It was only in 1982 that Al Neuharth persuaded Gannett's board to launch *USA Today*.

Creating a national newspaper in the United States was a logistical nightmare. Newspaper distribution is inherently a local business. That meant *USA Today* would have to be printed at plants across the country. With the Internet, that would have been straightforward. But in 1982, the only practical option was satellite transmission. With color pages, *USA Today* was a bleeding-edge technology.

Because we see those blue boxes nearly everywhere now, we tend to think that *USA Today* must have been a good idea. But just because something is successful today doesn't mean that it was worth the cost. It took Gannett twelve years before they broke even on the paper. Along the way, they lost over a billion dollars. And that was when a billion was real money.

If only Gannett had waited a few more years, the technology would have made their journey much easier. The problem was that the potential market for national papers in the United States was at most one. Neuharth was worried that Knight Ridder would launch first and then the window would be gone for good.

Both Apple and *USA Today* are cases where companies were playing a preemption game. The first person to launch has a chance to own the market, provided they succeed. The question is when to pull the trigger. Pull too early and you'll miss. Wait too long and you'll get beaten.

The way we describe a preemption game suggests a duel and that analogy is apt. If you fire too soon and miss, your rival will be able to advance and hit with certainty. But if you wait too long, you may end up dead without having fired a shot.* We can model the duel as an auction. Think of the time to shoot as the bid. The person who bids lowest gets the first chance to shoot. The only problem with bidding low is that the chance of success also goes down.

It might come as a surprise that both players will want to fire at the same time. That is to be expected when the two players have the same skill. But the result holds even when the two have different abilities.

Imagine that it were otherwise. Say you were planning to wait until time 10 before shooting. Meanwhile, your rival was planning to shoot at 8. That pair of strategies can't be an equilibrium. Your rival should change his strategy. He can now wait until time 9.99 and thereby increase his chance of success without risking being shot first. Whoever plans to go first should wait until the moment before the rival is about to shoot.

TRIP TO THE GYM NO. 8

Imagine that you and your rival both write down the time at which you will shoot. The chance of success at time t is $p(t)$ for you and $q(t)$ for your rival. If the first shot hits, the game is over. If it misses, then the other person waits to the end and hits with certainty. When should you shoot?

If waiting until time 10 really makes sense, you have to be willing to be shot at and hope your rival misses. That has to be every bit as good as jumping the gun and shooting first. The right time to fire is when your chance of success equals the rival's chance of failure. And since the chance of failure is 1 minus the chance of success, this implies that you fire the first moment when the two chances of success add up to 1. As you can see, if the two probabilities add up to 1 for you, they also add up to 1 for your rival. Thus the time to shoot is the same for both players. You get to prove this in our trip to the gym.

The way we modeled this game, both sides had correct understanding of the other side's chance of success. This might not always be true. We also assumed that the payoff from trying and failing was the same as the payoff from letting the other side go first and have it win. As they might say, sometimes it is better to have tried and lost than never to have tried at all.

THE WAR OF ATTRITION

The opposite of the preemption game is a war of attrition. Instead of seeing who jumps in first, here the objective is to outlast your rival. Instead of who goes

in first, the game is who gives in first. This, too, can be seen as an auction. Think of your bid as the time that you are willing to stay in the game and lose money. It is a bit of a strange auction in that all the participants end up paying their bid. The high bidder still wins. And here it may even make sense to bid more than your value.

In 1986, British Satellite Broadcasting (BSB) won the official license to provide satellite TV to the English market. This had the potential to be one of the most valuable franchises in all of history. For years, English TV viewers were limited in their choices to the two BBC channels and ITV. Channel 4 brought the total to, you guessed it, four. This was a country with 21 million households, high income, and plenty of rain. Moreover, unlike the United States, there was hardly any presence of cable TV.* Thus it was not at all unrealistic to imagine that the satellite TV franchise in the UK could bring in £2 billion revenue annually. Such untapped markets are few and far between.

Everything was looking up for BSB until June 1988, when Rupert Murdoch decided to spoil the fun. Working with an old-fashioned Astra satellite positioned over the Netherlands, Murdoch was able to beam his four channels to England. Now the Brits could finally enjoy *Dallas* (and soon *Baywatch*).

While the market might have seemed large enough for both Murdoch and BSB, the brutal competition between them destroyed all hopes for profit. They got into bidding wars over Hollywood movies and price wars over the cost of ad time. Because their broadcast technologies were incompatible, many people decided to wait and see who would win before investing in a dish.

After a year of competition, the two firms had lost a combined £1.5 billion. This was entirely predictable. Murdoch well understood that BSB wasn't going to fold. And BSB's strategy was to see if they could drive Murdoch into bankruptcy. The reason both firms were willing to suffer such massive losses is that the prize for winning was so large. If either one managed to outlast the other, it would have all of the profits to itself. The fact that you may have already lost £600 million is irrelevant. You've lost that amount whether you continue to play or give up. The only question is whether the additional cost of hanging on is justified by the pot of gold to the winner.

We can model this as an auction in which each side's bid is how long it will stay in the game, measured in terms of financial losses. The company that lasts longest wins. What makes this type of auction especially tricky is that there isn't any one best bidding strategy. If you think the other side is just about to fold, then you should always stay in another period. The reason you might think they are about to fold is because you think that they think you are going to stay in the game.

As you can see, your bidding strategy all depends on what you think they are doing, which in turn depends on what they think you are doing. Of course you don't actually know what they are doing. You have to decide in your head what they think you are up to. Because there's no consistency check, the two of you can each be overconfident about your ability to outlast the other. This can lead to massive overbidding or massive losses for both players.

Our suggestion is that this is a dangerous game to play. Your best move is to work out a deal with the other player. That's just what Murdoch did. At the eleventh hour, he formed a merger with BSB. The ability to withstand losses determined the split of the joint venture. And the fact that both firms were in danger of going under forced the government's hand in allowing the only two players to merge.

There's a second moral to this game: never bet against Murdoch.

CASE STUDY: SPECTRUM AUCTIONS

The mother of all auctions has been the sale of spectrum for cell phone licenses. Between 1994 and 2005, the Federal Communications Commission raised over \$40 billion. In England, an auction for 3G (third-generation) spectrum raised an eye-popping £22.5 billion, making it the biggest single auction of all time.¹

Unlike the traditional ascending bid auction, some of these auctions were more complicated because they allowed participants to simultaneously bid on several different licenses. In this case, we are going to give you a simplified version of the first U.S. spectrum auction and ask that you develop a bidding strategy. We'll see how you do relative to the actual auction participants.

In our stripped-down auction there will be just two bidders, AT&T and MCI, and just two licenses, NY and LA. Both firms are interested in both licenses, but there is only one of each.

One way to run the auctions would be to sell the two licenses in sequence. First NY and then LA. Or should it be first LA and then NY? There's no obvious answer as to which license should be sold first. Either way causes a problem. Say NY is sold first. AT&T might prefer LA to NY but feel forced to bid on NY knowing that winning LA is far from certain. AT&T would rather end up with something than nothing. But having won NY, it may not then have the budget to bid on LA.

With help from some game theorists, the FCC developed an ingenious solution to this problem: they ran a simultaneous auction. Both NY and LA were

up on the auction block at the same time. In effect, participants could call out their bids for either of the two licenses. If AT&T got outbid for LA, it could either raise its offer on LA or move on to bid for NY.

The simultaneous auction was over only when none of the bidders were willing to raise the price on any of the properties up for sale. In practice, the way this worked was that the bidding was divided up into rounds. Each round, players could raise or stay put.

We illustrate how this works using the example below. At the end of round 4, AT&T is the high bidder in NY, and MCI is the high bidder in LA.

	NY	LA
AT&T	6	7
MCI	5	8

In the bidding for round 5, AT&T could bid on LA, and MCI could choose to bid on NY. There's no point in AT&T bidding again on NY, as it is already the high bidder. Ditto for MCI and LA.

Imagine that only AT&T bids. In that case the new result might be:

	NY	LA
AT&T	6	9
MCI	5	8

Now AT&T is the high bidder on both properties. It can't bid. But the auction isn't over yet. The auction only ends when neither party bids in a round. Since AT&T bid in the previous round, there must be at least one more round, and MCI will have a chance to bid. If MCI doesn't bid, the auction is over. Remember that AT&T can't bid. If MCI does bid, say 7 for NY, then the auction continues. In the round that follows, AT&T could bid for NY, and MCI would have another chance to top the bid in LA.

The point of the above example was to make the rules of the auction clear. Now we will ask you to play the auction starting from scratch. To help you out, we'll share our market intelligence with you. The two firms spent millions of dollars preparing for the auction. As part of their preparation, they figured out both their own value for each of the licenses and what they thought their rival's might be. Here are the valuations:

	NY	LA
AT&T	10	9
MCI	9	8

According to the table above, AT&T values both licenses more than MCI. We want you to take this as a given. Furthermore, these valuations are known to both parties. AT&T not only knows its valuation, it knows MCI's numbers and knows that MCI knows AT&T's numbers and that MCI knows AT&T knows MCI's numbers, and so on and so forth. Everyone knows everything. Of course this is an extreme assumption, but the firms did spend a huge amount of money on what is called competitive intelligence, and so the fact that they had good knowledge about the other is pretty accurate.

Now you know the rules of the auction and all the valuations. Let's play. Since we are gentlemen, we'll let you pick which side to take. You picked AT&T? That's the right choice. They have the highest valuations, so you certainly have the advantage in this game. (If you didn't pick AT&T, would you mind picking again?)

It's time to place your bid(s). Please write them down. We've written our bid down and you can trust us to have made our bids without looking at what you've written.

Case Discussion

Before revealing our bid, let's consider some options you may have tried.

Did you bid 10 in NY and 9 for LA? If so, you've certainly won both auctions. But you've made no profit at all. This is one of the more subtle points about bidding in an auction. If you have to pay your bid—as you do in this case—then it makes little sense to bid your value. Think of this as akin to bidding \$10 to win a \$10 bill. The result is a wash.

The potential confusion here is that it may seem as if there is an extra prize from winning the auction, separate from what you win. Or, if you think of the valuation numbers as maximum bids, but not what you really think the item is worth, then again you might be happy to win at a bid equal to your value.

We don't want you to take either of these approaches. When we say your valuation is 10 for NY, what we mean by that is you are happy to walk away at 10 without whining or winning. At a price of 9.99 you would prefer to win, but only by a tiny amount. At a price of 10.01, you would prefer not to win, although the loss would be small.

Taking this perspective into account, you can see that bidding 10 for NY and 9 for LA is actually a case of a (weakly) dominated strategy. With this strategy, you are guaranteed to end up with zero. This is your payoff whether you win or lose. Any strategy that gives you a chance of doing better than zero while never losing any money will weakly dominate the strategy of bidding 10 and 9 right off the bat.

Perhaps you bid 9 in NY and 8 for LA. If so, you've certainly done better than bidding 10 and 9. Based on our bid, you'll win both auctions. (We won't bid more than our valuations.) So, congratulations.

How did you do? You made a profit of 1 in each city or 2 in total. The key question is whether you can do better.

You obviously can't do better bidding 10 and 9. Nor can you do better repeating your bids of 9 and 8. What other strategies might you consider? Let's assume that you bid 5 and 5. (The way the game will play out for other bids will be quite similar.) Now it's time for us to reveal our bid: we started with 0 (or no bid) in NY and 1 in LA. Given the way the first round of bidding has turned out, you are the high bidder in both cities. Thus you can't bid this round (as there is no point in having you top your own bid). Since we are losing out in both cities, we will bid again.

Think of the situation from our shoes. We can't go back home empty-handed to our CEO and say that we dropped out of the auction when the bids were at 5. We can only go home empty-handed if the prices have escalated to 9 and 8, so that it isn't worth our while to bid anymore. Thus we'll raise our bid in LA to 6. Since we just outbid you, the auction is extended another period. (Remember that the auction is extended another round whenever someone bids.) What will you do?

Imagine that you raise us in LA with a bid of 7. When it comes time for us to bid in the next round, we'll bid in NY this time with an offer of 6. We'd rather win NY at 6 than LA at 8. Of course, you can then outbid us back in NY.

You can see where this is all headed. Depending on who bids when, you will win both licenses at prices of 9 or 10 in NY and 8 or 9 in LA. This is certainly no better than the result when you just started out with a bid of 9 in NY and 8 in LA. It doesn't appear that our experiment has led to any improvement in payoffs. That happens. As you try out different strategies you can't expect them all to work. But was there something else you could have done that would have led to a profit greater than 2?

Let's go back and replay the last auction. What else might you have done after we bid 6 for LA? Recall that at that time, you were the high bidder in NY at a price of 5. Actually, you could have done nothing. You could have stopped

bidding. We had no interest in outbidding you in NY. We were plenty happy to win the LA license at a price of 6. The only reason we bid again is that we couldn't go away empty-handed—unless, of course, prices escalated to 9 and 8.

If you had stopped bidding, the auction would have ended then and there. You would only have won just one license, NY, at 5. Since you value that license at 10, this result is worth 5 to you, a big improvement over the gain of 2 you expect with bids of 9 and 8.

Think again from our perspective. We know that we can't beat you in both licenses. You have a higher valuation than we do. We are more than happy to walk away with a single license at any price we can below 9 and 8.

With all this practice, we should give you one last chance to bid and prove you really understand how this game works. Ready? Did you bid 1 in NY and 0 in LA? We hope so—because we bid 0 for NY and 1 for LA. At this point, we each have another chance to bid (as the bids from the previous round mean that the auction gets extended). You can't bid for NY, as you are already the high bidder. What about LA? Do you bid? We certainly hope...not. We didn't bid. So if you didn't bid, the auction is over. Remember that the auction ends as soon as there is a round with no bids. If the auction ends at that point, you walk away with just one license, but at the bargain price of 1, and thus you end up making 9.

It may be frustrating to have us win the second license at 1 when you value it well above that level and even more than we do. The following perspective might help soothe your spirits.

Before we walk away with no license, we will bid all the way up to 9 and 8. If you intend to deny us any license, you have to be prepared to bid a total of 17. Right now you have one license at a price of 1. Thus the true cost of winning the second license is 16, which is well in excess of your value.

You have a choice. You can win one license at a price of 1 or two licenses at a combined price of 17. Winning one is the better option. Just because you can beat us in both auctions doesn't mean that you should.

At this point, we'll bet that you still have some questions. For example, how would you know that we would be bidding on LA and leave you the opportunity to bid on NY? In truth, you wouldn't. We were lucky the way things worked out in this case. But, even if we had both bid on NY in the first round, it wouldn't have taken too long to sort things out.

You might also be wondering if this is collusion. Strictly speaking, the answer is no. While it is true that the two firms both end up better off (and the seller is the big loser), observe that neither party needs to make an agreement with the other. Each side is acting in its own best interest. MCI understands all

on its own that it can't win both licenses in the auction. This is no surprise, as AT&T has higher values for each item. Thus MCI will be happy to win either license. As for AT&T, it can appreciate that the true cost of the second license is the additional amount it will have to pay on both. Outbidding MCI on LA can raise the price in both LA and NY. The true cost of winning the second license is 16, more than its value.

What we see here is often called tacit cooperation. Each of the two players in the game understands the long-run cost of bidding for two licenses and thus recognizes the advantage of winning one license on the cheap. If you were the seller, you would want to avoid this outcome. One approach is to sell the two licenses in sequence. Now, it wouldn't work for MCI to let AT&T get the NY license for 1. The reason is that AT&T would still have every incentive to go after the LA license in the next auction. The key difference is that MCI can't go back and rebid in the NY auction, so AT&T has nothing to lose when bidding for the LA license.

The larger lesson here is that when two games are combined into one, this creates an opportunity to employ strategies that go across the two games. When Fuji entered the U.S. film market, Kodak had the opportunity to respond in the United States or in Japan. While starting a price war in the United States would have been costly to Kodak, doing so in Japan was costly to Fuji (and not to Kodak, who had little share in Japan). Thus the interaction between multiple games played simultaneously creates opportunities for punishment and cooperation that might otherwise be impossible, at least without explicit collusion.

Moral: If you don't like the game you are playing, look for the larger game.

For more auction case studies, have a look at chapter 14: "The Safer Duel," "The Risk of Winning," and "What Price a Dollar?"