

ষষ্ঠ অধ্যায়

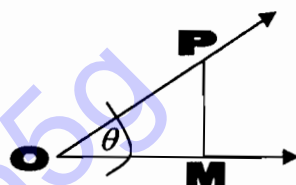
ত্রিকোণমিতি (Trigonometry)

সংজ্ঞা : 'ত্রি' অর্থ তিন আর 'মিতি' অর্থ পরিমাপ। সুতরাং 'ত্রিকোণমিতি' শব্দের অর্থ তিন কোণের পরিমাপ। তবে ত্রিকোণমিতি শুধু কোণের পরিমাপ নিয়েই আলোচনা করে না বরং তিন বাহু নিয়ে আলোচনা করে। সর্বোপরি বলা যায়, যে শাস্ত্র ত্রিভুজের তিন কোণ ও অন্যান্য বিষয়নিয়ে আলোচনা করে তাই ত্রিকোণমিতি। ত্রিকোণমিতির অনুপাত হলো ছয়টি। যথা : $\sin, \cos, \tan, \cot, \sec, \operatorname{cosec}$ ।

৬.১ ছয়টি ত্রিকোণমিতিক অনুপাতের সূত্র :

পার্শ্বের চিত্রটি লক্ষ্য করণ ,

চিত্রে $OP =$ অতিভুজ, $PM =$ লম্ব, $OM =$ ভূমি



$$\sin \theta = \frac{PM}{OP} = \frac{\text{লম্ব}}{\text{অতিভুজ}} \quad \operatorname{cosec} \theta = \frac{OP}{PM} = \frac{\text{অতিভুজ}}{\text{লম্ব}}$$

$$\cos \theta = \frac{OM}{OP} = \frac{\text{ভূমি}}{\text{অতিভুজ}} \quad \sec \theta = \frac{OP}{OM} = \frac{\text{অতিভুজ}}{\text{ভূমি}}$$

$$\tan \theta = \frac{PM}{OM} = \frac{\text{লম্ব}}{\text{ভূমি}} \quad \cot \theta = \frac{OM}{PM} = \frac{\text{ভূমি}}{\text{লম্ব}}$$

কোণ : α, β

বাস্তব সংখ্যা (একটি বিন্দুর স্থানাংক) : x, y

সমগ্র সংখ্যা : k

৬.২ কোণ পরিমাপের রেডিয়ান ও ডিগ্রী কোণের মধ্যে সম্পর্ক :

$$\text{৩৫৪. } 1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57^\circ 17' 45''$$

$$\text{৩৫৫. } 1^\circ = \frac{\pi}{180} \text{ rad} \approx 0.017453 \text{ rad}$$

$$৩৫৬. 1' = \frac{\pi}{180.60} \text{rad} \approx 0.000291 \text{rad}$$

$$৩৫৭. 1'' = \frac{\pi}{180.3600} \text{rad} \approx 0.000005 \text{rad}$$

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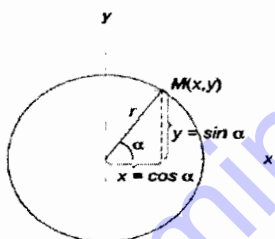
কোণ (ডিগ্রীতে)	0	30	45	60	90	180	270	360
কোণ (রেডিয়ানে)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

৬.৩ ত্রিকোণমিতিক অপেক্ষকের সংজ্ঞা ও লেখ :

$$৩৫৯. \sin \alpha = \frac{y}{r}$$

$$৩৬১. \tan \alpha = \frac{y}{x}$$

$$৩৬৩. \sec \alpha = \frac{r}{x}$$

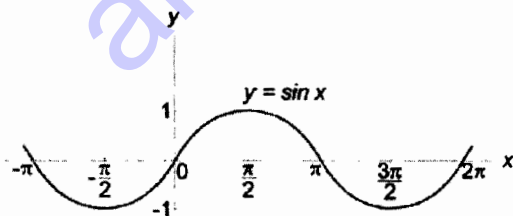


$$৩৬০. \cos \alpha = \frac{x}{r}$$

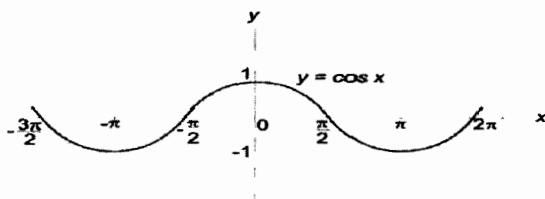
$$৩৬২. \cot \alpha = \frac{x}{y}$$

$$৩৬৪. \operatorname{cosec} \alpha = \frac{r}{y}$$

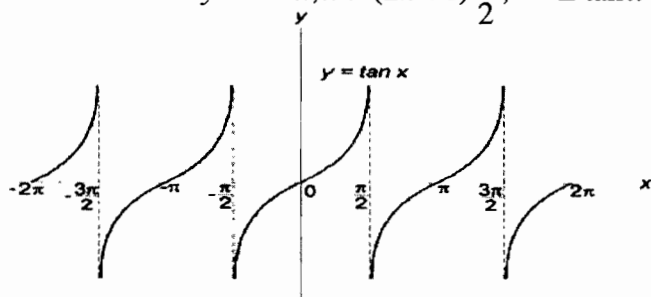
৩৬৫. সাইন অপেক্ষক : $y = \sin x, -1 \leq \sin x \leq 1$



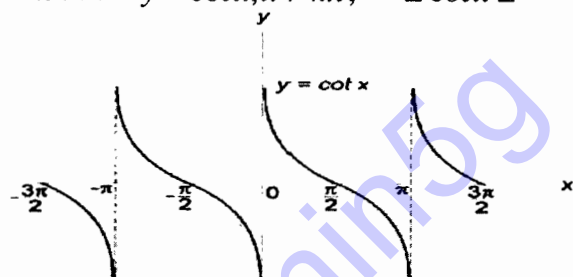
৩৬৬. কোসাইন অপেক্ষক : $y = \cos x, -1 \leq \cos x \leq 1$



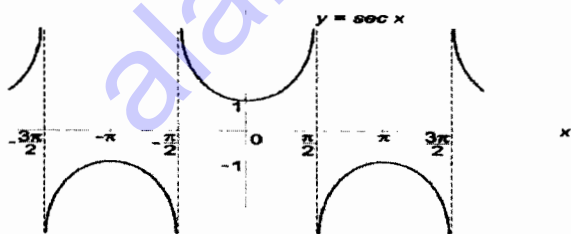
৩৬৭. ট্যানজেন্ট অপেক্ষক : $y = \tan x, x \neq (2k+1)\frac{\pi}{2}, -\infty \leq \tan x \leq \infty$



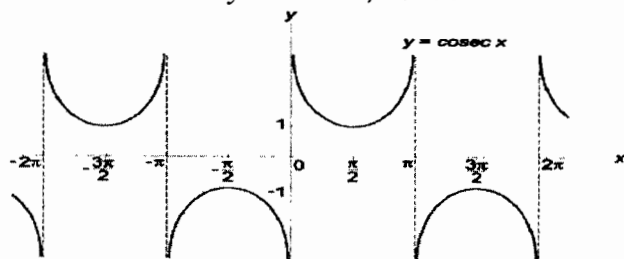
৩৬৮. কটজেন্ট অপেক্ষক : $y = \cot x, x \neq k\pi, -\infty \leq \cot x \leq \infty$



৩৬৯. সেকজেন্ট অপেক্ষক : $y = \sec x, x \neq (2k+1)\frac{\pi}{2}$



৩৭০. কোসাইনজেন্ট অপেক্ষক : $y = \operatorname{cosec} x, x \neq k\pi$

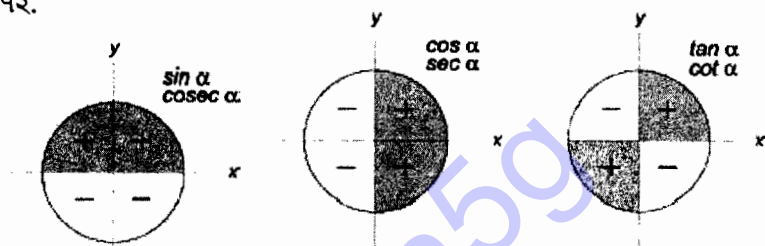


৬.৪ ত্রিকোণমিতিক অপেক্ষকের চিহ্ন :

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অক্ষ	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\operatorname{cosec} \alpha$
I	+	+	+	+	+	+
II	+					
III						
IV						

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৬.৫ বিভিন্ন কোণের জন্য ত্রিকোণমিতিক অপেক্ষকের মান :

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α°	α rad	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\operatorname{cosec} \alpha$
0	0	0	1	0	∞	1	∞
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90	$\frac{\pi}{2}$	1	0	∞	0	∞	1
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	-	$\frac{2}{\sqrt{3}}$
180	π	0	-1	0	∞	-1	∞

270	$\frac{3\pi}{2}$	-1	0	∞	0	∞	-1
360	2π	0	1	0	∞	1	∞

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α°	α rad	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$
15	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
36	$\frac{\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10+2\sqrt{5}}}$
54	$\frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{\sqrt{10+2\sqrt{5}}}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}+1}$
72	$\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
75	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$

৬.৬ ত্রিকোণমিতির গুরুত্বপূর্ণ সূত্র সমূহ :

৩৭৫. $\sin^2 \alpha + \cos^2 \alpha = 1$

৩৭৬. $\sec^2 \alpha - \tan^2 \alpha = 1$

৩৭৭. $\operatorname{cosec}^2 \alpha - \cot^2 \alpha = 1$

৩৭৮. $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$

৩৭৯. $\cot \alpha = \frac{\cos \alpha}{\sin \alpha}$

৩৮০. $\tan \alpha \cdot \cot \alpha = 1$

৩৮১. $\sec \alpha = \frac{1}{\cos \alpha}$

$$৩৮২. \operatorname{cosec} \alpha = \frac{1}{\sin \alpha}$$

৬.৭ লঘুকরণ সূত্র :

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β	$\sin \beta$	$\cos \beta$	$\tan \beta$	$\cot \beta$
$-\alpha$	$-\sin \alpha$	$+\cos \alpha$	$-\tan \alpha$	$-\cot \alpha$
$90^\circ - \alpha$	$+\cos \alpha$	$+\sin \alpha$	$+\cot \alpha$	$+\tan \alpha$
$90^\circ + \alpha$	$+\cos \alpha$	$-\sin \alpha$	$-\cot \alpha$	$-\tan \alpha$
$180^\circ - \alpha$	$+\sin \alpha$	$-\cos \alpha$	$-\tan \alpha$	$-\cot \alpha$
$180^\circ + \alpha$	$-\sin \alpha$	$-\cos \alpha$	$+\tan \alpha$	$+\cot \alpha$
$270^\circ - \alpha$	$-\cos \alpha$	$-\sin \alpha$	$+\cot \alpha$	$+\tan \alpha$
$270^\circ + \alpha$	$-\cos \alpha$	$+\sin \alpha$	$-\cot \alpha$	$-\tan \alpha$
$360^\circ - \alpha$	$-\sin \alpha$	$+\cos \alpha$	$-\tan \alpha$	$-\cot \alpha$
$360^\circ + \alpha$	$+\sin \alpha$	$+\cos \alpha$	$+\tan \alpha$	$+\cot \alpha$

৬.৮ ত্রিকোণমিতির পর্যায়বৃত্ত অপেক্ষক :

$$৩৮৪. \sin(\alpha \pm 2\pi) = \sin \alpha, \text{ পর্যায় } 2\pi \text{ বা, } 360^\circ$$

$$৩৮৫. \cos(\alpha \pm 2\pi) = \cos \alpha, \text{ পর্যায় } 2\pi \text{ বা, } 360^\circ$$

$$৩৮৬. \tan(\alpha \pm \pi) = \tan \alpha, \text{ পর্যায় } \pi \text{ বা, } 180^\circ$$

$$৩৮৭. \cot(\alpha \pm \pi) = \cot \alpha, \text{ পর্যায় } \pi \text{ বা, } 180^\circ$$

৬.৯ ত্রিকোণমিতিক অপেক্ষকের মধ্যে সম্পর্ক :

$$৩৮৮. \sin \alpha = \pm \sqrt{1 - \cos^2 \alpha} = \pm \sqrt{\frac{1}{2}(1 - \cos 2\alpha)} = 2 \cos^2 \left(\frac{\alpha}{2} - \frac{\pi}{4} \right) - 1$$

$$= \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$৩৮৯. \cos \alpha = \pm \sqrt{1 - \sin^2 \alpha} = \pm \sqrt{\frac{1}{2}(1 + \cos 2\alpha)} = 2 \cos^2 \frac{\alpha}{2} - 1$$

$$= \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$৩৯০. \tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \pm \sqrt{\sec^2 \alpha - 1} = \frac{\sin 2\alpha}{1 + \cos 2\alpha} = \frac{1 - \cos 2\alpha}{\sin 2\alpha}$$

$$= \pm \sqrt{\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}} = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$৩৯১. \cot \alpha = \frac{\cos \alpha}{\sin \alpha} = \pm \sqrt{\operatorname{cosec}^2 \alpha - 1} = \frac{1 + \cos 2\alpha}{\sin 2\alpha} = \frac{\sin 2\alpha}{1 - \cos 2\alpha}$$

$$= \pm \sqrt{\frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}} = \frac{1 - \tan^2 \frac{\alpha}{2}}{2 \tan \frac{\alpha}{2}}$$

$$৩৯২. \sec \alpha = \frac{1}{\cos \alpha} = \pm \sqrt{1 + \tan^2 \alpha} = \frac{1 + \tan^2 \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}}$$

$$৩৯৩. \operatorname{cosec} \alpha = \frac{1}{\sin \alpha} = \pm \sqrt{1 + \cot^2 \alpha} = \frac{1 + \tan^2 \frac{\alpha}{2}}{2 \tan \frac{\alpha}{2}}$$

৬.১০ ত্রিকোণমিতিক যোগ-বিয়োগের সূত্র :

$$৩৯৪. \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$৩৯৫. \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha$$

$$৩৯৬. \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$৩৯৭. \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$৩৯৮. \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$৩৯৯. \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$৪০০. \cot(\alpha + \beta) = \frac{1 - \tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$$

$$৪০১. \cot(\alpha - \beta) = \frac{1 + \tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$$

৬.১১ দ্বিকোণের সূত্র :

$$৪০২. \sin 2\alpha = \sin \alpha \cdot \cos \alpha$$

$$৪০৩. \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2\sin^2 \alpha = 2\cos^2 \alpha - 1$$

$$৪০৪. \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2}{\cot \alpha - \tan \alpha}$$

$$৪০৫. \cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha} = \frac{\cot \alpha - \tan \alpha}{2}$$

৬.১২ গুণিত কোণের সূত্র :

$$৪০৬. \sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha = 3\cos^2 \alpha \cdot \sin \alpha - \sin^3 \alpha$$

$$৪০৭. \sin 4\alpha = 4\sin \alpha \cdot \cos \alpha - 8\sin^3 \alpha \cdot \cos \alpha$$

$$৪০৮. \sin 5\alpha = 5\sin \alpha - 20\sin^3 \alpha + 16\sin^5 \alpha$$

$$৪০৯. \cos 3\alpha = 4\cos^3 \alpha - 3\cos \alpha = \cos^3 \alpha - 3\cos \alpha \cdot \sin^2 \alpha$$

$$৪১০. \cos 4\alpha = 8\cos^4 \alpha - 8\cos^2 \alpha + 1$$

$$৪১১. \cos 5\alpha = 16\cos^5 \alpha - 20\cos^3 \alpha + 5\cos \alpha$$

$$৪১২. \tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}$$

$$৪১৩. \tan 4\alpha = \frac{4 \tan \alpha - 4 \tan^3 \alpha}{1 - 6 \tan^2 \alpha + \tan^4 \alpha}$$

$$৪১৪. \tan 5\alpha = \frac{\tan^5 \alpha - 10 \tan^3 \alpha + 5 \tan \alpha}{1 - 10 \tan^2 \alpha + 5 \tan^4 \alpha}$$

$$৪১৫. \cot 3\alpha = \frac{\cot^3 \alpha - 3 \cot \alpha}{3 \cot^2 \alpha - 1}$$

$$816. \cot 4\alpha = \frac{1 - 6\tan^2 \alpha + \tan^4 \alpha}{4\tan \alpha - 4\tan^3 \alpha}$$

$$819. \cot 5\alpha = \frac{1 - 10\tan^2 \alpha + 5\tan^4 \alpha}{\tan^5 \alpha - 10\tan^3 \alpha + 5\tan \alpha}$$

৬.১৩ অর্ধ কোণের সূত্র :

$$818. \sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$819. \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$820. \tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha} = \operatorname{cosec} \alpha - \cot \alpha$$

$$821. \cot \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}} = \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha} = \operatorname{cosec} \alpha + \cot \alpha$$

৬.১৪ অর্ধ কোণের ট্যানজেন্ট পরিচয় :

$$822. \sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$823. \cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$824. \tan \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}}$$

$$825. \cot \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{2 \tan \frac{\alpha}{2}}$$

৬.১৫ অন্যান্য ত্রিকোণমিতিক সূত্র সমূহ :

$$826. \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$827. \sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$828. \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$879. \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$880. \tan \alpha + \tan \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$$

$$881. \tan \alpha - \tan \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta}$$

$$882. \cot \alpha + \cot \beta = \frac{\sin(\beta + \alpha)}{\sin \alpha \sin \beta}$$

$$883. \cot \alpha - \cot \beta = \frac{\sin(\beta - \alpha)}{\sin \alpha \sin \beta}$$

$$884. \cos \alpha + \sin \alpha = \sqrt{2} \cos \left(\frac{\pi}{4} - \alpha \right) = \sqrt{2} \sin \left(\frac{\pi}{4} + \alpha \right)$$

$$885. \cos \alpha - \sin \alpha = \sqrt{2} \sin \left(\frac{\pi}{4} - \alpha \right) = \sqrt{2} \cos \left(\frac{\pi}{4} + \alpha \right)$$

$$886. \tan \alpha + \cot \beta = \frac{\cos(\alpha - \beta)}{\cos \alpha \sin \beta}$$

$$887. \tan \alpha - \cot \beta = \frac{\cos(\alpha + \beta)}{\cos \alpha \sin \beta}$$

$$888. 1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}$$

$$889. 1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2}$$

$$890. 1 + \sin \alpha = 2 \cos^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

$$891. 1 - \sin \alpha = 2 \sin^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

$$892. \sin \alpha \sin \beta = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$

$$893. \cos \alpha \cos \beta = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2}$$

$$894. \sin \alpha \cos \beta = \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2}$$

$$৪৪৫. \tan \alpha \cdot \tan \beta = \frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta}$$

$$৪৪৬. \cot \alpha \cdot \cot \beta = \frac{\cot \alpha + \cot \beta}{\tan \alpha + \tan \beta}$$

$$৪৪৭. \tan \alpha \cdot \cot \beta = \frac{\tan \alpha + \cot \beta}{\cot \alpha + \tan \beta}$$

৬.১৬ ত্রিকোণমিতিক অপেক্ষকের শক্তি :

$$৪৪৮. \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} \quad ৪৪৯. \sin^3 \alpha = \frac{3\sin \alpha - \sin 3\alpha}{4}$$

$$৪৫০. \sin^4 \alpha = \frac{\cos 4\alpha - 4\cos 2\alpha + 3}{8}$$

$$৪৫১. \sin^5 \alpha = \frac{10\sin \alpha - 5\sin 3\alpha + \sin 5\alpha}{16}$$

$$৪৫২. \sin^6 \alpha = \frac{10 - 15\cos 2\alpha + 6\cos 4\alpha - \cos 6\alpha}{32}$$

$$৪৫৩. \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2} \quad ৪৫৪. \cos^3 \alpha = \frac{3\cos \alpha + \cos 3\alpha}{4}$$

$$৪৫৫. \cos^4 \alpha = \frac{\cos 4\alpha + 4\cos 2\alpha + 3}{8}$$

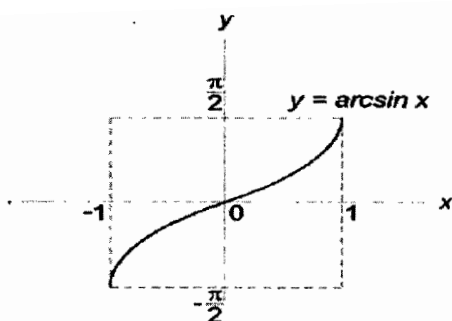
$$৪৫৬. \cos^5 \alpha = \frac{10\cos \alpha + 5\sin 3\alpha + \cos 5\alpha}{16}$$

$$৪৫৭. \cos^6 \alpha = \frac{10 + 15\cos 2\alpha + 6\cos 4\alpha + \cos 6\alpha}{32}$$

৬.১৭ ত্রিকোণমিতিক বিপরীত অপেক্ষকের লেখ :

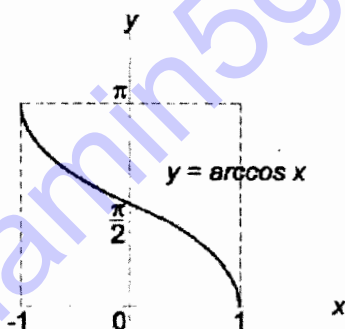
৪৫৮. বিপরীত সাইন অপেক্ষক :

$$y = \arcsin x, -1 \leq x \leq 1, -\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$$



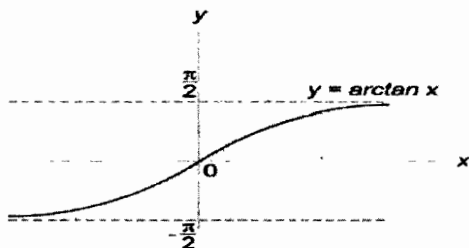
৪৫৯. বিপরীত কোসাইন অপেক্ষক :

$$y = \arccos x, -1 \leq x \leq 1, 0 \leq \arccos x \leq \pi$$



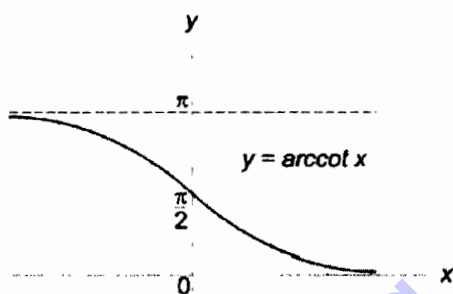
৪৬০. বিপরীত ট্যানজেন্ট অপেক্ষক :

$$y = \arctan x, -\infty \leq x \leq \infty, -\frac{\pi}{2} \leq \arctan x \leq \frac{\pi}{2}$$



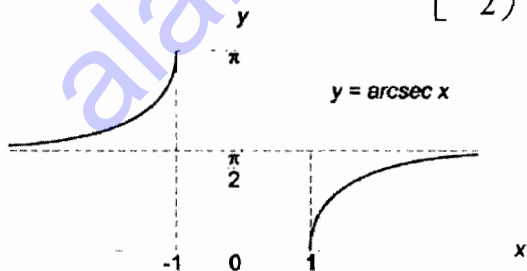
৪৬১. বিপরীত কটজেন্ট অপেক্ষক :

$$y = \operatorname{arccot} x, -\infty \leq x \leq \infty, 0 \leq \operatorname{arccot} x \leq \pi$$



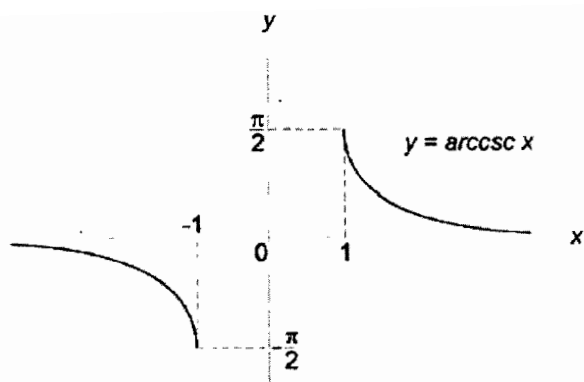
৪৬২. বিপরীত সেকজেন্ট অপেক্ষক :

$$y = \operatorname{arcsec} x, x \in (-\infty, -1] \cup [1, \infty), \operatorname{arcsec} x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$$



৪৬৩. বিপরীত কোসেকজেন্ট অপেক্ষক :

$$y = \operatorname{arccosec} x, x \in (-\infty, -1] \cup [1, \infty), \operatorname{arccosec} x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$$



৬.১৮ ত্রিকোণমিতিক বিপরীত অপেক্ষকের মধ্যে সম্পর্ক :

$$868. \arcsin(-x) = -\arcsin x$$

$$869. \arcsin x = \frac{\pi}{2} - \arccos x$$

$$870. \arcsin x = \arccos \sqrt{1-x^2}, 0 \leq x \leq 1$$

$$871. \arcsin x = -\arccos \sqrt{1-x^2}, -1 \leq x \leq 0$$

$$872. \arcsin x = \arctan \frac{x}{\sqrt{1-x^2}}, x^2 < 1.$$

$$873. \arcsin x = \operatorname{arc cot} \frac{\sqrt{1-x^2}}{x}, 0 < x \leq 1.$$

$$874. \arcsin x = \operatorname{arc cot} \frac{\sqrt{1-x^2}}{x} - \pi, -1 \leq x \leq 0.$$

$$875. \arccos(-x) = \pi - \arccos x$$

$$876. \arccos x = \frac{\pi}{2} - \arcsin x$$

$$877. \arccos x = \arcsin \sqrt{1-x^2}, 0 \leq x \leq 1$$

$$878. \arccos x = \pi - \arcsin \sqrt{1-x^2}, -1 \leq x \leq 0$$

$$894. \arccos x = \arctan \frac{\sqrt{1-x^2}}{x}, 0 < x \leq 1.$$

$$895. \arccos x = \pi + \arctan \frac{\sqrt{1-x^2}}{x}, -1 \leq x < 0.$$

$$896. \arccos x = \operatorname{arccot} \frac{x}{\sqrt{1-x^2}}, -1 \leq x \leq 1.$$

$$897. \arctan(-x) = -\arctan x$$

$$898. \arctan x = \frac{\pi}{2} - \operatorname{arccot} x$$

$$899. \arctan x = \arcsin \frac{x}{\sqrt{1+x^2}}$$

$$900. \arctan x = \arccos \frac{1}{\sqrt{1+x^2}}, x \geq 0.$$

$$901. \arctan x = -\arccos \frac{1}{\sqrt{1+x^2}}, x \leq 0.$$

$$902. \arctan x = \frac{\pi}{2} - \arctan \frac{1}{x}, x > 0.$$

$$903. \arctan x = -\frac{\pi}{2} - \arctan \frac{1}{x}, x < 0.$$

$$904. \arctan x = \operatorname{arccot} \frac{1}{x}, x > 0.$$

$$905. \arctan x = \operatorname{arccot} \frac{1}{x} - \pi, x < 0.$$

$$906. \operatorname{arccot}(-x) = \pi - \operatorname{arccot} x$$

$$907. \operatorname{arccot} x = \frac{\pi}{2} - \arctan x$$

$$908. \operatorname{arccot} x = \arcsin \frac{1}{\sqrt{1+x^2}}, x > 0.$$

$$8৯০. \operatorname{arc} \cot x = \pi - \arcsin \frac{1}{\sqrt{1+x^2}}, x < 0.$$

$$8৯১. \operatorname{arc} \cot x = \arccos \frac{x}{\sqrt{1+x^2}}$$

$$8৯২. \operatorname{arc} \cot x = \arctan \frac{1}{x}, x > 0.$$

$$8৯৩. \operatorname{arc} \cot x = \pi + \arctan \frac{1}{x}, x < 0.$$

৬.১৯ ত্রিকোণমিতিক সমীকরণ : সমগ্র সংখ্যা : n

$$8৯৪. \sin x = a, x = (-1)^n \arcsin a + \pi n$$

$$8৯৫. \cos x = a, x = \pm \arccos a + 2\pi n$$

$$8৯৬. \tan x = a, x = \arctan a + \pi n$$

$$8৯৭. \cot x = a, x = \operatorname{arc} \cot a + \pi n$$

৬.২০ পরাবৃত্তীয় অপেক্ষকের মধ্যে সম্পর্ক : কাল্পনিক সংখ্যা : i

$$8৯৮. \sin(ix) = i \sinh x$$

$$8৯৯. \tan(ix) = i \tanh x$$

$$৫০০. \cot(ix) = -i \coth x$$

$$৫০১. \sec(ix) = \sec hx$$

$$৫০২. \operatorname{cosec}(ix) = -i \operatorname{cosech} x$$